

1.

A tank contains 125 gallons of heating oil at time  $t = 0$ . During the time interval  $0 \leq t \leq 12$  hours, heating oil is pumped into the tank at the rate

$$H(t) = 2 + \frac{10}{(1 + \ln(t+1))} \text{ gallons per hour.}$$

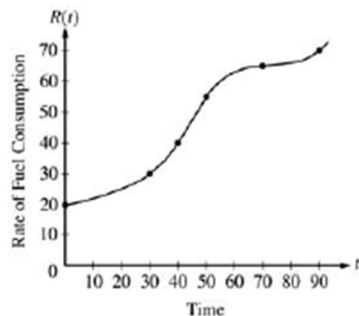
During the same time interval, heating oil is removed from the tank at the rate

$$R(t) = 12 \sin\left(\frac{t^2}{47}\right) \text{ gallons per hour.}$$

- How many gallons of heating oil are pumped into the tank during the time interval  $0 \leq t \leq 12$  hours?
- Is the level of heating oil in the tank rising or falling at time  $t = 6$  hours? Give a reason for your answer.
- How many gallons of heating oil are in the tank at time  $t = 12$  hours?
- At what time  $t$ , for  $0 \leq t \leq 12$ , is the volume of heating oil in the tank the least? Show the analysis that leads to your conclusion.

2.

The rate of fuel consumption, in gallons per minute, recorded during an airplane flight is given by a twice-differentiable and strictly increasing function  $R$  of time  $t$ . The graph of  $R$  and a table of selected values of  $R(t)$ , for the time interval  $0 \leq t \leq 90$  minutes, are shown above.



$t$ (minutes)	$R(t)$ (gallons per minute)
0	20
30	30
40	40
50	55
70	65
90	70

- Use data from the table to find an approximation for  $R'(45)$ . Show the computations that lead to your answer. Indicate units of measure.
- The rate of fuel consumption is increasing fastest at time  $t = 45$  minutes. What is the value of  $R''(45)$ ? Explain your reasoning.
- Approximate the value of  $\int_0^{90} R(t) dt$  using a left Riemann sum with the five subintervals indicated by the data in the table. Is this numerical approximation less than the value of  $\int_0^{90} R(t) dt$ ? Explain your reasoning.
- For  $0 < b \leq 90$  minutes, explain the meaning of  $\int_0^b R(t) dt$  in terms of fuel consumption for the plane. Explain the meaning of  $\frac{1}{b} \int_0^b R(t) dt$  in terms of fuel consumption for the plane. Indicate units of measure in both answers.

3.

The wind chill is the temperature, in degrees Fahrenheit ( $^{\circ}\text{F}$ ), a human feels based on the air temperature, in degrees Fahrenheit, and the wind velocity  $v$ , in miles per hour (mph). If the air temperature is  $32^{\circ}\text{F}$ , then the wind chill is given by  $W(v) = 55.6 - 22.1v^{0.16}$  and is valid for  $5 \leq v \leq 60$ .

- Find  $W'(20)$ . Using correct units, explain the meaning of  $W'(20)$  in terms of the wind chill.
- Find the average rate of change of  $W$  over the interval  $5 \leq v \leq 60$ . Find the value of  $v$  at which the instantaneous rate of change of  $W$  is equal to the average rate of change of  $W$  over the interval  $5 \leq v \leq 60$ .
- Over the time interval  $0 \leq t \leq 4$  hours, the air temperature is a constant  $32^{\circ}\text{F}$ . At time  $t = 0$ , the wind velocity is  $v = 20$  mph. If the wind velocity increases at a constant rate of 5 mph per hour, what is the rate of change of the wind chill with respect to time at  $t = 3$  hours? Indicate units of measure.